Light Weight Structures – Application of Topology Optimization Using Stress Limit as a Criteria in Formulation

Anuj Anand[1], Siddesh Kapdi[2], M D Jinto[3] and Akash Bhuwal[4]

[1,2,3,4]Eaton, Modeling and Simulation, Pune, MH, India
anujanand@eaton.com

Abstract

Topology optimization has evolved as one of the preferred techniques used in the concept stage of a product development cycle across the industry. The most commonly used approach in topology optimization for a static load case is the stiffness or compliance-based classical approach. Incorporating stress limits in the optimization formulation is still one of the major challenges in topology optimization.

To make topology optimization more robust and efficient, it is critical to address stress, given its effect on the failure criteria and product life directly. This paper discusses different problem formulations in topology optimization for static load cases and the ways it addresses the stress limit.

To illustrate the methods, engineering examples are subjected to optimization formulation and the results are verified through finite element analysis. The aim of this study is to understand the application of topology optimization to design light-weight structures subjected to stress limits using commercial optimization solvers.

Keywords: *Topology optimization, Stress constraint, L shape, MBB and FEA*

1 Introduction

Topology optimization is a mathematical technique which gives an optimum material layout for a defined problem formulation. This optimization technique has a wide range of industrial applications and has emerged as an effective tool for developing light-weight structures. It is used in the early stages of a design process where the design flexibility is comparatively high.

A commonly followed approach involves performing optimization in the concept stage with general structural parameters such as stiffness, weight etc., and evaluating the durability parameters later in the design stage. Addressing the durability parameters within the topology optimization can effectively help reduce the design cycle time. This can be achieved by introducing stress as a constraint in optimization.

Topology optimization could provide material layout in terms of solids and voids over a design domain, like a checker board, but still satisfying the problem formulation. The optimal topologies generated by this way are difficult to manufacture since many regions with voids are involved. As a substitute method, the power law approach which is also known as the Solid Isotropic Material with Penalization (SIMP) is used so that the material distribution problem can be solved [1].

Mathematical statement of the density-based topology optimization problem comprises an objective function, constraints and discretized representation of physical system. A general formulation based on linear static finite element analysis can be represented as [2]:

$$\begin{aligned} \text{Min:} \quad & f(\rho, U) \\ \text{subjected to:} \quad & K(\rho)\, U = F(\rho) \\ & g_i(\rho, U) \le 0 \\ & 0 \le \rho \le 1 \end{aligned} \tag{1}$$

where $\boldsymbol{f}$ is the objective function, ρ is the vector of density design variables, **U** is the displacement vector, **K** is the global stiffness matrix, **F** is the load vector and g_i are the constraints. With generalized statement, the number of problem formulations can be formulated from different physics such as compliance, displacements, frequencies and contact gap, among others, for structural domain.

SIMP is an interpolation scheme that interpolates the young's modulus of a material to the scalar selected field using power law. In other words, the SIMP approach replaces the integer variables with continuous variables and then introduces some form of penalty that gives the solution to discrete 0/1 values [3].

$$E = \rho_i^p E_0 \tag{2}$$

where ρ_i is the relative density of element
$\boldsymbol{p}$ is penalty factor,
$\boldsymbol{E_0}$ and **E** are elastic modulus before and after penalty respectively.
The stiffness matrix is given by

$$K = K(X) = \sum_i^n \rho_i^p E_0 K_i \tag{3}$$

where $\boldsymbol{K_i}$ is element stiffness matrix.

The topology optimization work flow of a classical formulation for structural problems is shown in Figure 1, where the stiffness matrix is penalized as shown in equation (3), density plots are interpreted in CAD and verified through FEA for durability. If the durability requirements are not met in the design validation checks, it may require more modifications and validations which is an iterative process leading to increased design cycle time. Techniques such as DOE or shape optimization are also used to meet the design/stress criteria or fine tune the design variables after the topology optimization.

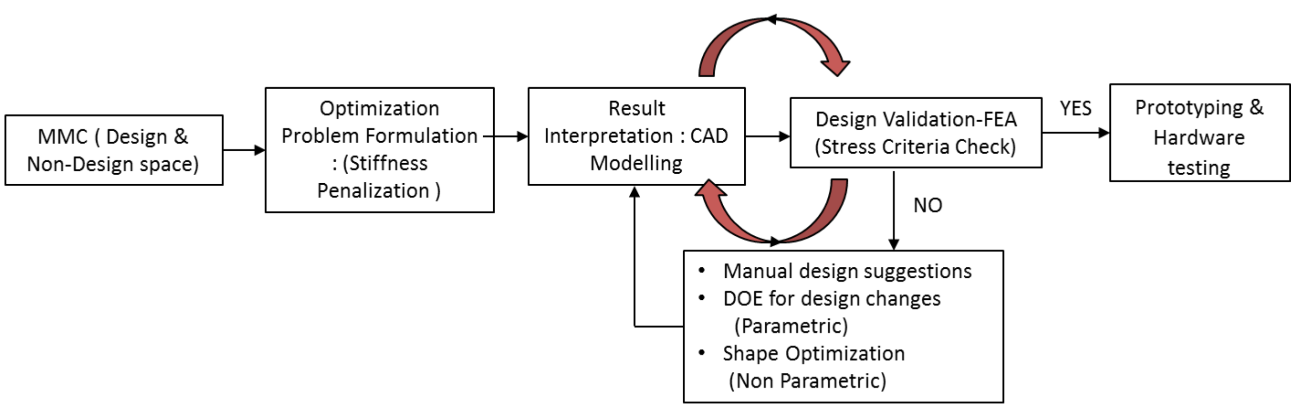

Figure 1 Work Flow of Topology Optimization

Minimize compliance or maximize stiffness with a mass constraint is applied for structures instead of using the stress criteria in most engineering applications. The purpose of addressing stress in the optimization formulation stage is to avoid high stress concentrations and control the stress levels in early stages of the design process. To implement stress as a constraint in topology optimization, there were two basic challenges—singularity phenomena and the local nature of stress constraints.

In order to account for stress in the optimization problem formulation stress penalization method is discussed below which originates from Bruggi [4]. It gives penalization of intermediate design variables and also avoids the singularity problem. The singularity problem is discussed in many papers, such as Kirsch [5] Rozvany and Birker [6] and to avoid singularity, one must use a relaxation approach as suggested by Cheng and Guo [7]. The relaxation approach was successfully used by Duysinx, Bendsoe [8], Duysinx and Sigmund [9], where two approaches were used, which have global and local stress constraints, respectively. In local stress constraints, one stress constraint is used in each stress evaluation point of each element and by global, it means that one stress constraint is used for the entire model.

This paper covers stress penalization technique in Section 2 and three different formulations in Section 3. Sections 4 and 5 present the optimization results of an L-shaped geometry and an MBB beam. Section 6 contains concluding remarks with recommendations and future perceptions.

2 Stress Penalization

The objective of the penalization technique is to achieve a final design without intermediate design variables. It was discussed in the so-called qp-approach [10] where a stress penalization was used with another exponent instead of stiffness penalization.

Similar penalization technique was used by Le et al. [11] but with a fixed exponent. The stress penalization for design variable x_e, η_s where $\eta_S(\rho_e(x))$ is inserted as a SIMP penalization function [12]

$$\eta_S(\rho_e(x)) = (\rho_e(x))^{1/2} \tag{4}$$

Where x_e and ρ_e represents the e:th design variable and e:th filtered variable

The solid material stress tensor for stress evaluation point a, as expressed in Voigt notation,

$$\hat{\sigma}_a(x) = (\hat{\sigma}_{ax}\ \hat{\sigma}_{ay}\ \hat{\sigma}_{az}\ \hat{\tau}_{axy}\ \hat{\tau}_{ayz}\ \hat{\tau}_{azx})^T \tag{5}$$

and it is calculated by the finite element analysis (FE-analysis), as

$$\hat{\sigma}_a(x) = E\, B_a\, u(x), \tag{6}$$

where E is the constitutive matrix,

B_a is the strain-displacement matrix corresponding to stress evaluation point a $(\hat{\sigma}_a(x))$

u(x) is the global vector of nodal displacements

The penalized stress tensor for stress evaluation point a then reads

$$\hat{\sigma}_a(x) = \eta_S(\rho_e(x))\, \hat{\sigma}_a(x) \tag{7}$$

3 Problem Formulation

An L-shaped geometry and an MBB beam are the structures considered for the evaluation of the problem formulations. Structures are discretized into finite elements with each element having its own element density ($x_1..x_n$) as the design variable for the topology optimization. For a static load case problem, the finite element equation is written as

$$K(\rho(x))u = F \tag{8}$$

where ρ(x) is the filtered design variable created by taking a weighted average of adjacent design variables, K(ρ(x)) is the global stiffness matrix of the structure, u is the global nodal displacement vector and F is a vector of external loads [12].

In the first P_1 optimization problem formulation, the classical approach is followed with minimized compliance as the objective function and as a constraint.

P_1 formulation:

$$\text{Objective: } \min_x \frac{1}{2} F^T u(x) \tag{9}$$

$$\text{Constraint: } \sum_{e=1}^{n_e} m_e \rho_e(x) \le \bar{M}$$

Where $\bar{M}$ is the allowable total mass and n_e is the number of design variables, m_e is the corresponding element mass and e is the design variable number associated to a particular element. In this particular study, the allowable mass limit ($\bar{M}$) was 0.5 times the original mass of the structure.

In the second problem formulation, along with the classical approach, a stress constraint is added in the formulation [12].

P_2 formulation:

$$\text{Objective: } \min_x \frac{1}{2} F^T u(x) \tag{10}$$

$$\text{Constraint: } \sum_{e=1}^{n_e} m_e \rho_e (x) \le \bar{M}$$

$$\text{Constraint: } \sigma_i (x) \le \bar{\sigma}$$

In the third problem formulation, compliance is used neither as an objective nor as a constraint. Minimize the topology element mass is used as the objective function and stress is used as the only constraint in this case [12].

P_3 formulation:

$$\text{Objective: } \min_x \sum_{e=1}^{n_e} m_e \rho_e (x) \tag{11}$$

$$\text{Constraint: } \sigma_i (x) \le \bar{\sigma}$$

Optimization density plots of all the three formulations were compared to understand the effect of stress constraint in the problem formulation.

4 L-Shaped Geometry

L-shaped geometry is the most used example for topology optimization to study the effect of stress as a constraint. As from the literature, the L-shaped geometry contains a corner with a geometric stress singularity. Dimensions of the L-shaped geometry are as shown in Figure 3. The material considered is aluminum with mechanical properties as seen in Table 1.

Table 1 Material Details

Material properties of Aluminum	
Elastic Modulus	7.8 E10 Pa
Poisson's ratio	0.33
Density	2800 kg/m^3
Yield strength	300 E6 Pa

FE analysis was performed using ANSYS Workbench v 15.0[13]. In order to simulate the L-shaped geometry under static load, elements with appropriate properties were defined in the FE model. A full model of the geometry is presented in Figure 2. A static load of 1500 N is applied at one end of the L beam and the other end of the L beam is fixed.

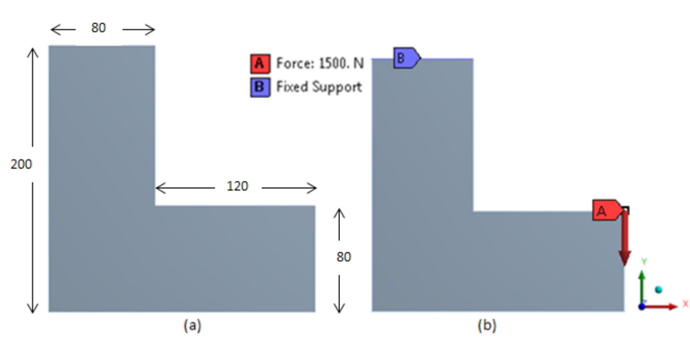

Figure 2 L shape beam (a) Geometry (b) Finite element Model

Figure 3 presents the contour plots for equivalent von-Mises stress and total deformation. The total deformation at L shape beam is 0.003 m Figure 4. Solution for P_1 formulation (a) Element Density plot (b) von-Mises stress plot and the von-Mises stress is around 512 Mpa at the corner of the beam. The maximum stress value is at the location where point load is applied and the stress value at the corner is not converged due to singularity or sharp corner

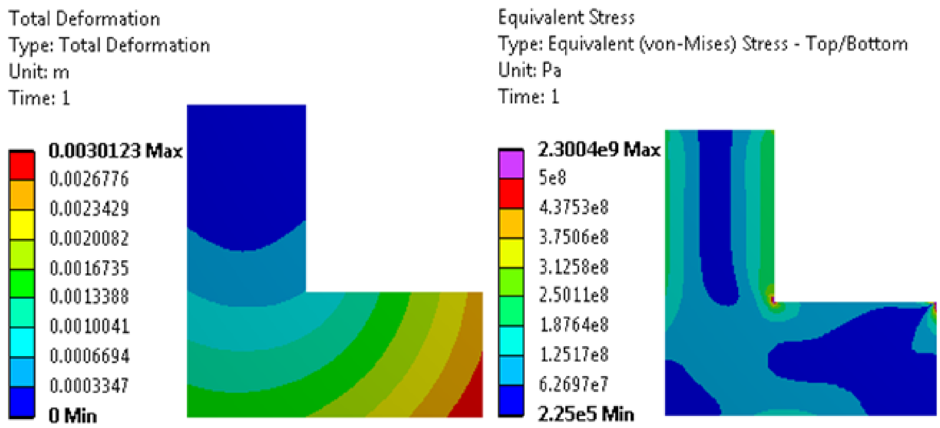

Figure 3 FE analysis results (a) Total deformation (b) von-Mises stress plot

Figure 4(a) shows the results of P_1 formulation, the element density contour plot at threshold >0.5 and von Mises stress of optimized shape [14] and [15]. In the L-shape geometry, no internal radius is formed to prevent from a stress peak in the corner using classical formulation in Optistruct [14]. The value of von Mises stresses in Figure 4 (b) at the corner is not converged due to sharp corner or a singularity point.

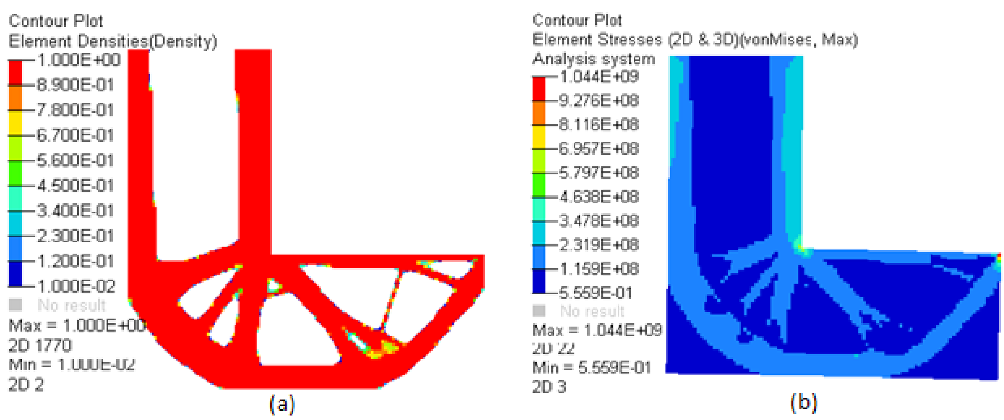

Figure 4 Solution for P_1 formulation (a) Element Density plot (b) von-Mises stress plot

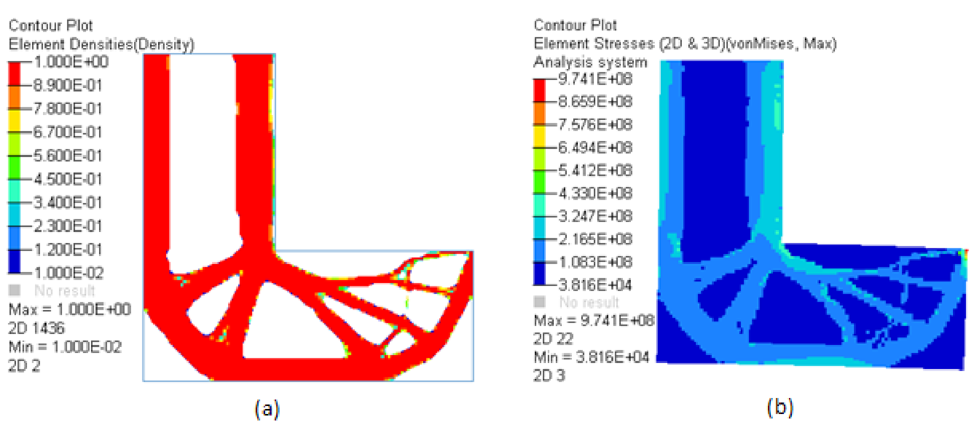

Figure 5 Solution for P_2 formulation (a) Element Density plot (b) von-Mises stress plot

On the other hand, the element density plots for P_2 formulation give a lighter design as compared with P_1 formulation. It also creates a radius at the corner of the L-shaped geometry to avoid the stress concentration in the internal corner as shown in Figure 5(a). The value of von Mises stress in Figure 5(b) is converged as the sharp corner is avoided in this formulation result [14] and [15].

Problem formulation P_3, where objective has changed from maximize stiffness to minimize mass with stress as a constraint, gives a smooth radius as seen in element density plot of Figure 6(a) to avoid the stress concentration in the internal corner of the L-shaped geometry. The value of von Mises stress in Figure 6(b) is converged. As the sharp corner is avoided by a smooth radius and stress value is also reduced in this formulation results as compared with P_2 formulation [14] and [15].

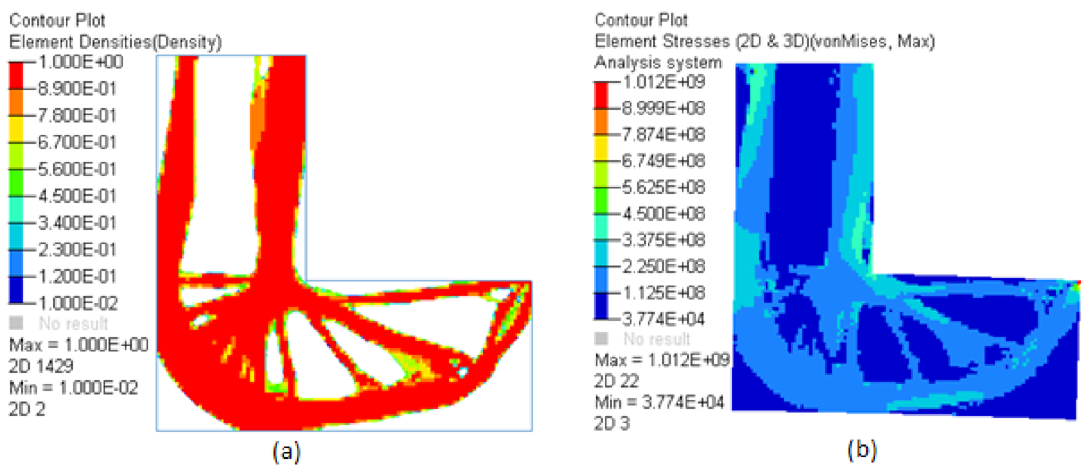

Figure 6 Solution for P_3 formulation (a) Element Density plot (b) von-Mises stress plot

For an L-shaped geometry, the more feasible results were obtained using P_3 formulation where minimize mass with stress as constraint gives smooth radius at the corner to avoid stress singularity and the optimal solution in terms of weight.

5 Messerschmitt-Bölkow-Blohm (MBB) - Beam

The MBB beam is also a commonly used example for topology optimization. As shown in figure 7(a), an MBB beam of dimension 600 mm×100 mm×1 mm has been selected for the topology optimization study [12]. Material properties used for MBB beam is same as that of the L-shaped geometry. The beam is meshed with uniform sized hex elements of size 2.5 mm. In order to take advantage of the symmetry of the MBB beam, a symmetric boundary condition is used as shown in figure 7 (b). A point load of magnitude 750N is applied on the top left node of the meshed model.

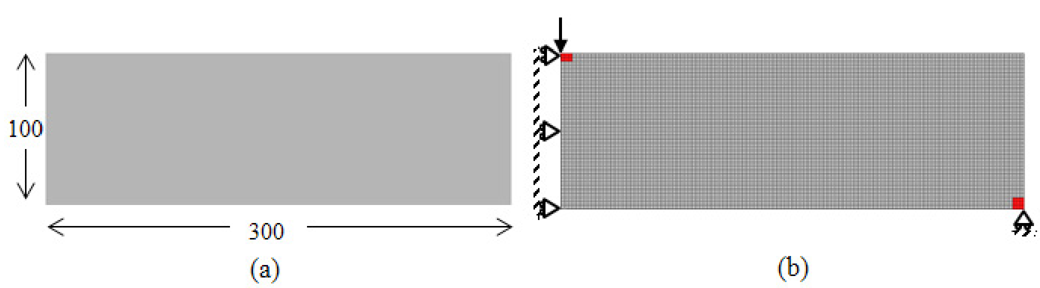

Figure 7 MBB Beam (a) Geometry (b) FEA model (non-design space elements are in red color)

An FE analysis is performed on the MBB beam using ANSYS Workbench v 15 [13] and total deformation plot and the equivalent von-Mises stress plot is shown in Figure 8 (a) and 8(b). Stress singularities were observed on those elements where nodal point load is applied and nodal constraint is given.

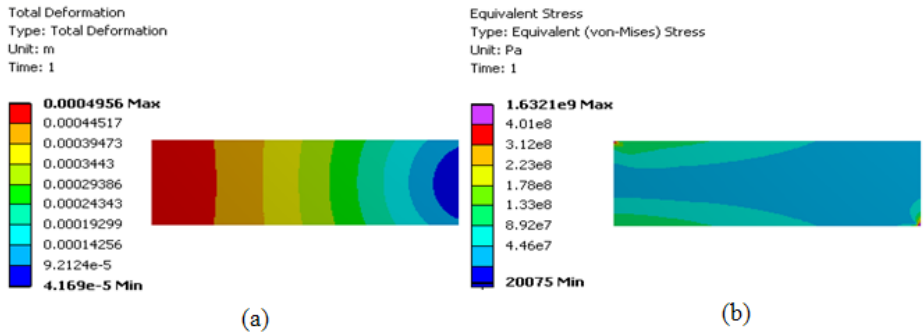

Figure 8 FE Analysis results for MBB Beam (a) Total deformation (b) von-Mises

Elements immediately surrounding these stress singularity points were considered in the non-design space for the topology optimization in order to avoid numerical singularity.

Similar to the case of L-shaped geometry, three different formulations (P_1, P_2 and P_3) are also used for the topology optimization of the MBB beam.

In P_1 formulation, the classical approach is used with a mass constraint where $\bar{M}$ in equation 9 is 0.5 times the original design space mass. Figure 9(a) shows the element density plot obtained for P_1 classical formulation at 0.5 threshold value [14] and [15]. In the case of topology optimization with classical formulation, stress singularities do not affect the optimized element density layout. Similar optimized density plots were observed irrespective of keeping the elements with stress singularities in design space or in non-design space. Equivalent von-Mises stress plot of the optimized material layout is shown in Figure 9 (b).

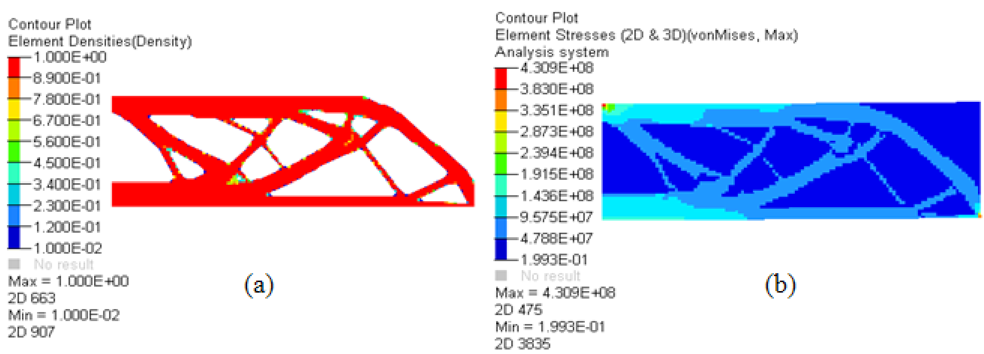

Figure 9 Solution for P_1 formulation (a) Element Density Plot (b) von-Mises stress

In the P_2 problem formulation for topology optimization, stress is used as an additional constraint along with the classical formulation. Element density plot obtained using topology optimization at 0.5 threshold value is shown in Figure 10(a) [14] and [15]. Since both the P_1 and P_2 formulations has used the same mass constraints, the final mass of the optimized material layout obtained in both cases are same. Figure 10(b) shows the von-Mises stress plot of the optimized material layout obtained through FE analysis and there is no significant change observed between the P_1 and P_2 formulations.

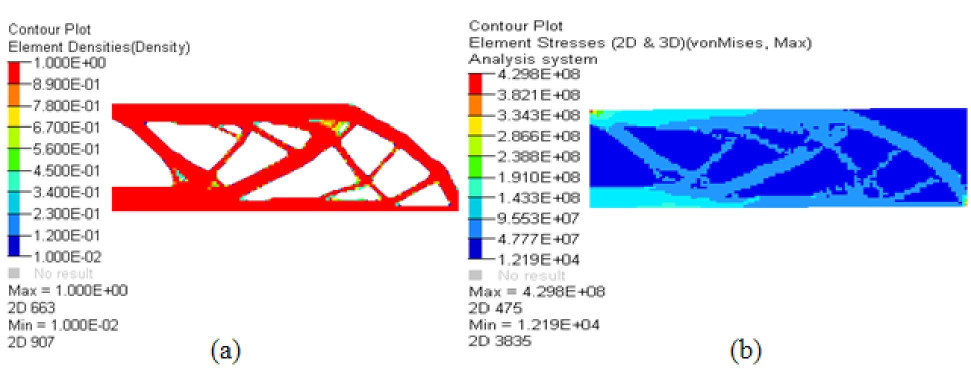

Figure 10 Solution for P_2 formulation (a) Element Density Plot (b) von-Mises stress

In P_3 problem formulation, instead of stiffness maximization, minimization of design space mass is used as the objective function and stress is used as a constraint. Figure 11(a) shows the element density plot obtained at 0.5 threshold value [14] and [15]. It is evident from the density plot that optimized material layout obtained using P_3 formulation is significantly different from what obtained using P_1 and P_2 formulations. Material layout obtained is considerably lighter when mass is used as an objective function instead of as a constraint.

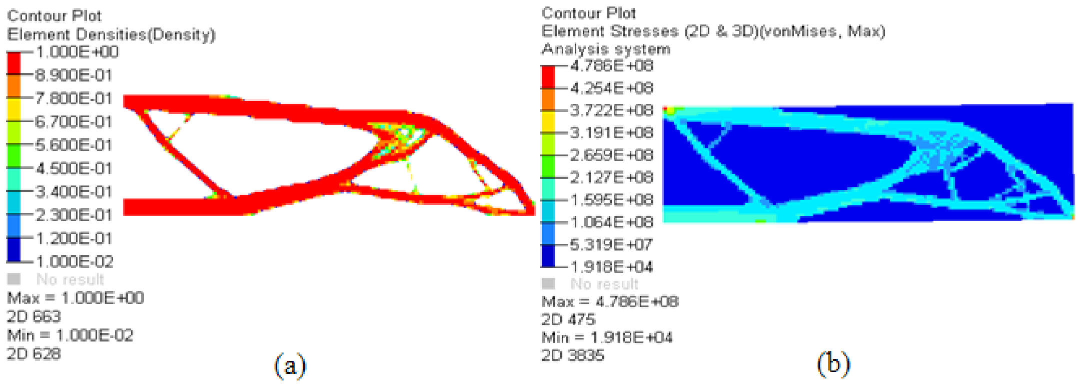

Figure 11 Solution for P_3 formulation (a) Element Density Plot (b) von-Mises stress

In the von-Mises stress plot of the optimized geometry shown in Figure 11 (b), it can be observed that if the stress singularities are excluded, stress distribution over the structure is uniform to a certain extent.

Conclusion

Fatigue life is one of the important parameters in durability. Thus, including stress in the problem formulation of topology optimization is important to reduce the design cycle time. In this paper, we evaluated three different formulations on two engineering examples: L-shaped geometry and MBB beam. A classical formulation with stiffness as objective function (P_1) is used in the first case along with a mass constraint. In the second formulation (P_2), the effect of an additional stress constraint is evaluated along with the classical formulation. Third formulation (P_3) replaces stiffness with mass as the objective function along with stress constraint. It was observed that P_2 and P_3 formulations offer important advantages over P_1 formulation by addressing stress concentrations in topology optimization. The same was evident in the L-shaped geometry example where sharp corners were replaced by smooth radius. Considering mass as the objective function, the P_3 formulation has an additional advantage of obtaining the lightest structure compared to other two formulations. Study on the MBB beam concludes that P_3 formulation gives a nearly uniform distribution of stress in the material layout at least in the case of simple geometries.

The study shows that incorporating stress as a constraint in problem formulation can effectively address the durability in the concept phase and similar investigation is needed on more complex geometries from real-life engineering applications.

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